Susie Lanier and Sharon Barrs

Activities

Let's Play Plinko: A Lesson in Simulations and Experimental Probabilities

FANS OF THE TELEVISION GAME SHOW *THE PRICE Is Right* call Plinko their favorite pricing game. Although traveling to the CBS studios in Los Angeles, California, to be a contestant on *The Price Is Right* is only a dream for most students, the exciting game of Plinko can be used to teach students such mathematics as simulations and experimental probabilities.

NCTM's *Principles and Standards for School Mathematics* (2000) supports this idea. The discussion of the Curriculum Principle suggests that the mathematics curriculum "should offer experiences that allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena" (NCTM 2000, pp. 17–18). Furthermore, the Data Analysis and Probability Standard expects high school students to be able to use simulations to construct empirical probability distributions.

Like many games, Plinko immediately appeals to students. It involves a real-life situation, competition, and money—all of which students can relate to. Teachers are attracted to Plinko, as well, because they can use it to present various mathematical topics. Haws (1995) used her version of a Plinko board to discuss probabilities and relationships to Pascal's triangle. Lemon (1997) developed counting strategies when she described the paths that chips follow on her Plinko board. Lemon's board modeled the board used on *The Price Is Right*.

The authors of this article have used Plinko to address such topics as counting paths, tree diagrams, equally likely and unequally likely events,

Plinko involves a real-life situation, competition, and money

experimental and theoretical probabilities, and simulations. In this article, we describe how we used Plinko to teach a group of high school students about simulations and experimental probabilities. These students were juniors and seniors in an advanced mathematics course that met for ninety-minute-long sessions. They were completing a unit on probability and game theory.

BRIEF DESCRIPTION OF PLINKO

The Plinko board is a maze consisting of rows of pegs. A contestant on *The Price Is Right* plays a minimum of one to a maximum of five chips, depending on the outcome of a pricing game that is played before Plinko. The contestant chooses a slot, and the chip enters the maze through that slot. When a chip hits an interior peg, it has an equal chance of falling to the left or to the right, except when it hits the side of the board or a peg on the outer boundary. If it hits the outer boundary, the chip falls in a direction that allows it to remain in play on the board. Chips come to rest in slots labeled with dollar amounts, at the bottom of the board, as follows: $100, $500, $1000, $0, $10,000, $0, $1000, $500, and $100. The largest amount of money ever won with Plinko was $23,000. Even students who are not familiar

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with the Plinko game can quickly catch on after a brief demonstration.

THE FIRST SIMULATION: PLAYING PLINKO ON THE PHYSICAL BOARD

Most students were familiar with *The Price Is Right,* but not necessarily with the Plinko game. To introduce the game to the students, we built our own board, shown in **figure 1.** First, we analyzed a videotape of the game to determine the design and dimensions for our board. Next, we obtained the necessary materials and constructed the board. We used a physical board to make the game seem more real and to motivate the students in an interactive and competitive activity.

Materials

- 3/16" × 4' × 8' white pegboard (enough for two Plinko boards)
- 1/4" × 1 1/4" grooved dowel pins (at least 153; we used 203)
- poker chips (at least 5)
- rubber bands (12 large, 1/4" wide)

Building the Plinko board

To build the board, the teacher first cuts the pegboard in half. The Plinko game area is centered on the board, as shown in **figure 1.** Using twenty-five rows by nineteen columns, the teacher places thir-
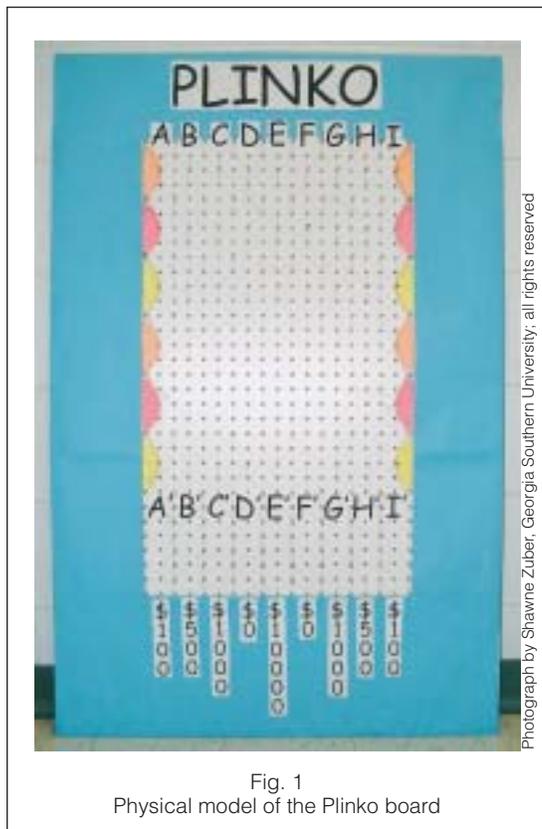
Fig. 1
Physical model of the Plinko board

teen rows of pegs, alternating ten and nine pegs per row, with an empty row between, as follows:

Peg row 1. Starting at the first hole, the teacher inserts ten pegs, skipping a hole between pegs. The teacher skips the next row of holes.

Peg row 2. The teacher skips the first hole, inserts nine pegs, skipping a hole between pegs, and skips a row of holes.

Peg rows 3, 5, 7, 9, 11, and 13. These rows are the same as row 1.

Peg rows 4, 6, 8, 10, and 12. These rows are the same as row 2.

We suggest making a trough in which the chips can land. We made six more rows of ten pegs, skipping a hole between each peg, but not skipping a row of holes. We then had nine columns at the bottom of the board. Our final row was a row of nineteen pegs.

The teacher next loops a rubber band around the first pegs in rows 1, 2, and 3 to create a triangle, which forms the border, or walls, of the board and then loops a rubber band on the last pegs in the same rows.

The teacher repeats this step with rows (3, 4, and 5), (5, 6, and 7), (7, 8, and 9), (9, 10, and 11), and (11, 12, and 13).

The next step is putting the title and dollar amounts on the board. We included the *A* and *A'* labels to make talking about the board easier. These labels can be painted or taped on the board; however, tape may hinder the play of the chip if it is not carefully applied. Also, the border of the board and the interiors of the triangular boundaries can be painted.

Once we had our board, we were ready to play. Our first activity was to drop a chip on the Plinko board and allow students to observe what happened. To create a competitive atmosphere, we divided the students into groups of five. Each student dropped a chip from a slot of his or her choice. A member of the class recorded the data on **sheet 1** and added together the winnings for each of the students in a group to obtain a team total. Each group enthusiastically tried to beat the other groups.

After everyone had a chance to play, we discussed the strategies that students had used while playing the game. The predominant question considered was, Where should I place my chip to give me the best chance of winning the big money? We did not discuss this question with the students before they played the game. We wanted them to play and then reflect on their actions. Many questions arose at this point. Did students randomly drop the chip? Why did they choose a certain starting slot? Is one starting slot better than another? How are the

Why does the experimental probability differ from the theoretical probability?

results influenced by the board's having only one $10,000 slot but two of each of the other slots? Why is the $10,000 slot surrounded by $0 slots? Does this arrangement affect where students should place their chip?

The discussion led to the ultimate question: What are my chances of winning $10,000? At that point, we defined such terms as *trials, favorable outcomes,* and *probability*. Our definitions are consistent with those of Musser and Burger (1997). We also discussed the importance of being able to model real-world situations and the relationship between experimental and theoretical probabilities. We used the class data and **sheet 2** to find the experimental probability for winning each of the dollar amounts shown on the board. We included theoretical probabilities on the sheet for comparison. We explained to students that the theoretical probabilities were calculated using techniques that

Where should I place my chip to give me the best chance of winning big money?

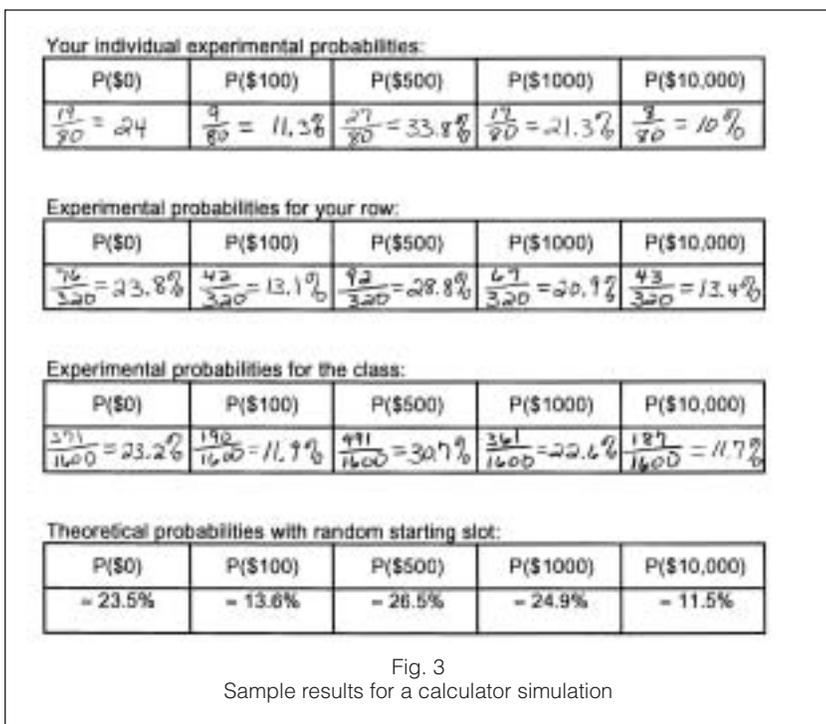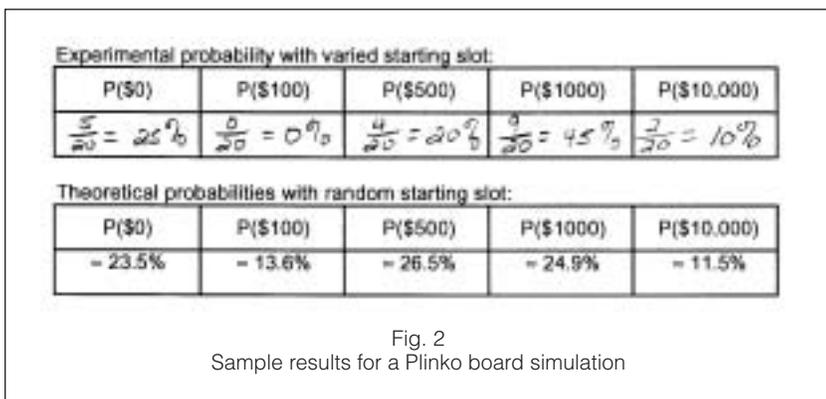the class would discuss in a future lesson. The students willingly accepted these values.

Figure 2 shows the results for a possible board simulation. During two of the twenty trials, the chip landed in the $10,000 slot. Hence, the experimental probability of winning $10,000, denoted $P(\$10,000)$, was 10 percent, compared to the theoretical probability of 11.5 percent. We then asked several questions: Why does our experimental probability differ from the theoretical probability? Are we satisfied with our results? Can we obtain results that are closer to the theoretical probability? Students commented that the board was symmetrical. They noticed that they avoided the outer slots (A, H, and I) and therefore did not randomly select a starting slot. They also suggested that getting more trials could produce a "better" value for the experimental probability. This discussion led to our using a programmable calculator to play Plinko.

THE SECOND SIMULATION: USING A CALCULATOR PROGRAM

With only twenty students in the class, we had a very small number of trials and were not satisfied with our experimental probabilities. Although each student could have dropped more than one chip, that method is time-consuming. Using a programmable calculator allows us to obtain thousands of trials in a short amount of time.

We wrote a Plinko program, given as **program 1,** for the TI-83 calculator. This program can be downloaded from the National Science Foundation (NSF) project "Demos with Positive Impact" Web site (Lanier and Barrs). In addition, a program for the TI-89 calculator is available on the same Web site. The program allows the user to select a specific starting slot or let the calculator randomly select the starting slots. Also, the user enters the number of chips to be dropped. We used **sheets 3a** and **3b** with this simulation. The students find three experimental probabilities: individual, small group, and class. **Figure 3** shows an example of results for a calculator simulation. In our classroom, each row of students formed a small group. The "your data" section on sheet **3a** asks each student to complete a total of eighty trials, a number that was selected because of the time factor. However, students can enter a larger number of trials at the beginning of class, let the calculators run throughout the class period, and tabulate data at the end of the class.

The individual experimental $P(\$10,000)$ ranged from 6 percent to 25 percent. Students in each row combined their data, for a total of 320 trials, and found that the row experimental $P(\$10,000)$ ranged from 9.7 percent to 13.4 percent. We then combined the row data. The total number of trials for the class was 1600, with a class experimental $P(\$10,000)$ of 11.7 percent. Of course, we want students to

Experimental probability with varied starting slot:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
$\frac{5}{20} = 25\%$	$\frac{0}{20} = 0\%$	$\frac{4}{20} = 20\%$	$\frac{9}{20} = 45\%$	$\frac{2}{20} = 10\%$

Theoretical probabilities with random starting slot:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
≈ 23.5%	≈ 13.6%	≈ 26.5%	≈ 24.9%	≈ 11.5%

Fig. 2
Sample results for a Plinko board simulation

Your individual experimental probabilities:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
$\frac{19}{80} = 24$	$\frac{9}{80} = 11.3\%$	$\frac{27}{80} = 33.8\%$	$\frac{17}{80} = 21.3\%$	$\frac{8}{80} = 10\%$

Experimental probabilities for your row:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
$\frac{76}{320} = 23.8\%$	$\frac{42}{320} = 13.1\%$	$\frac{92}{320} = 28.8\%$	$\frac{67}{320} = 20.9\%$	$\frac{43}{320} = 13.4\%$

Experimental probabilities for the class:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
$\frac{371}{1600} = 23.2\%$	$\frac{190}{1600} = 11.9\%$	$\frac{491}{1600} = 30.7\%$	$\frac{361}{1600} = 22.6\%$	$\frac{187}{1600} = 11.7\%$

Theoretical probabilities with random starting slot:

P($0)	P($100)	P($500)	P($1000)	P($10,000)
≈ 23.5%	≈ 13.6%	≈ 26.5%	≈ 24.9%	≈ 11.5%

Fig. 3
Sample results for a calculator simulation

understand the mathematical idea that the more trials that are used, the better the opportunity is for the experimental probability to more closely approximate the theoretical probability of 11.5 percent.

Other types of technology can be used to play Plinko. Computers can speed up the process of producing large numbers of trials and can provide users with a visual picture of the simulation. The "Demos with Positive Impact" Web site (Lanier and Barrs) contains a Java-script computer simulation. The activity sheets in this article can be adapted for these simulations.

A FINAL COMMENT ON THEORETICAL PROBABILITIES

We did not ask the students to calculate the theoretical probabilities during this activity. The actual calculations merit an entire lesson of their own. The task is not as simple as it may first appear. Because of the boundary on each side of the board, the paths that a chip may follow are not equally likely. The authors of this article originally used probability trees to calculate the probabilities. However, this task was cumbersome and time-consuming because of the size of the board (nine columns, thirteen rows). An easier and more exciting method for finding the probabilities uses Pascal's triangle. The Demos with Positive Impact Web site includes a discussion of a smaller version of a Plinko board. With our advanced students, we followed the simulation activity with a lesson on calculating the theoretical probabilities. We encourage readers of this article to explore finding these probabilities.

CONCLUSION

We accomplished several goals with these activities. The students experienced mathematics by simulating real-world phenomena. They developed an understanding of the importance of experimental probabilities and their relationship to theoretical probabilities. Perhaps most important, they had fun with mathematics. We hope that they will continue to develop a positive attitude about mathematics.

The students enjoyed both the board simulation and the calculator simulation. They were very competitive with one another. The experimental probability of winning $10,000 on a single chip was 10 percent for the board simulation and 11.7 percent for the calculator simulation. The students were able to observe that the calculator simulation (with 1580 more trials) did produce a better approximation of the theoretical probability (11.5 percent) of winning the $10,000. Thus, they experienced a variety of simulations and drew conclusions about experimental and theoretical probabilities. They left the classroom "feeling good" about mathematics. Several of the students were observed playing Plinko on their calculators during subsequent classes.

```
PROGRAM 1

Fix 0                          [B](I,P)+1→[B](I,P)
ClrHome                        End
Disp "NUMBER OF TRIALS"        For(I,2,18,2)
Prompt N                       [B](13,I)→L₂(I/2)
Disp "STARTING SLOT?"          End
Disp "ENTER 1-9 FOR"           End
Disp "SLOTS A-I or 0"
Disp "FOR RANDOM SLOT"         ClrHome
Input S                        Output(1,1,"A")
                               Output(1,3,L₂(1))
ClrList L₂                     Output(1,10,"F")
(13,19)→dim([B])               Output(1,12,L₂(6))
Fill(0,[B])                    Output(2,1,"B")
                               Output(2,3,L₂(2))
For(J,1,N)                     Output(2,10,"G")
{1,12}→dim([A])                Output(2,12,L₂(7))
ClrList L₁                     Output(3,1,"C")
randInt(0,1,12)→L₁             Output(3,3,L₂(3))
For(I,1,12)                    Output(3,10,"H")
If L₁(I)=0                     Output(3,12,L₂(8))
Then                           Output(4,1,"D")
-1→[A](1,I)                    Output(4,3,L₂(4))
Else                           Output(4,10,"I")
1→[A](1,I)                     Output(4,12,L₂(9))
End                            Output(5,1,"E")
End                            Output(5,3,L₂(5))
If S=0                         100*L₂(1)+500*L₂(2)+1000*L₂
Then                           (3)+0*L₂(4)+10000*L₂(5)+0*L
2*randInt(1,9)→P               ₂(6)+1000*L₂(7)+500*L₂(8)+1
Else                           00*L₂(9)→V
2*S→P                          Output(7,1,"AMT")
End                            Output(7,5,V)

[B](1,P)+1→[B](1,P)
For(I,2,13)
P+[A](1,I-1)→P
```

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(Worksheets begin on page 630.)

Name	Chip	Start	End	Dollar Amount	Winnings
	1				
	2				
	3				
	4				
	5				
	1				
	2				
	3				
	4				
	5				
	1				
	2				
	3				
	4				
	5				
	1				
	2				
	3				
	4				
	5				
	1				
	2				
	3				
	4				
	5				

Refer to the player chart on sheet 1 to complete the tables. Compute probabilities to the nearest 0.1 percent.

Total number of chips played: _____

Number of times a chip is dropped from slot:

A	B	C	D	E	F	G	H	I

Were the starting slots random? Explain.

Number of times a chip lands in slot:

A'	B'	C'	D'	E'	F'	G'	H'	I'

Number of times chip lands in dollar amount:

$0	$100	$500	$1000	$10,000

Experimental probabilities with varied starting slot:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)

Theoretical probabilities with random starting slot:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)
≈ 23.5%	≈ 13.6%	≈ 26.5%	≈ 24.9%	≈ 11.5%

Compare the experimental probabilities to the theoretical probabilities. Are they the same? If not, why not?

Stage 1

Run your Plinko program four times. Enter 20 as your number of trials and 0 to randomize your starting slot (see sample screen at right). Record your data from your result screen in the chart. You do not need to record the dollar amount.

```
PrgmPLINKO          A 3     F 1
NUMBER OF TRIALS    B 5     G 2
N=?20               C 3     H 1
STARTING SLOT?      D 2     I 1
ENTER 1-9 FOR       E 2
SLOTS A-I OR 0
FOR RANDOM          AMT 28400
?0
```
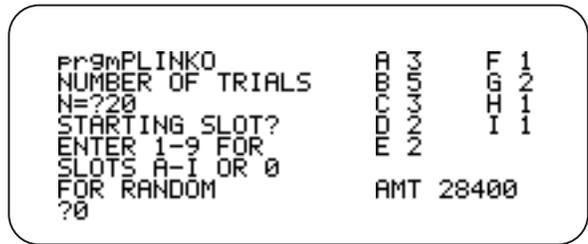
Your data: Random starting slot

	Number of Trials	A	B	C	D	E	F	G	H	I
	20									
	20									
	20									
	20									
Totals	80									

Stage 2

Combine your data with the data obtained by the other students in your group.

Your group's data: Random starting slot

	Number of Trials	A	B	C	D	E	F	G	H	I
Totals										

Stage 3

Combine the data for your group with the data obtained by all the other groups in the class.

Class data: Random starting slot

	Number of Trials	A	B	C	D	E	F	G	H	I
Totals										

From the *Mathematics Teacher,* December 2003

Use the data on the previous sheet to find the following experimental probabilities to the nearest 0.1 percent for each of the three stages.

Your individual experimental probabilities:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)

Experimental probabilities for your group:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)

Experimental probabilities for the class:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)

Theoretical probabilities with random starting slot:

P($0)	*P*($100)	*P*($500)	*P*($1000)	*P*($10,000)
≈ 23.5%	≈ 13.6%	≈ 26.5%	≈ 24.9%	≈ 11.5%

1. Compare your experimental probabilities to the theoretical probabilities. Are they the same? Are they closer than in the previous simulation? Why?

2. If you repeated the experiment with the same number of trials, would you obtain the same results? Why?

3. Make a conjecture about a simulation with 1,000,000 trials.